Student Workbook

Exponential and Logarithmic Equations

Name: ______________________________ Date: _______________ Class: ________

ESSENTIAL QUESTION

Why do we take the logarithm of both sides to solve an exponential equation — and why must we always check solutions to logarithmic equations for extraneous roots?

Lesson Overview

Exponential and logarithmic equations are inverses of each other — solving one type often requires converting to the other. Two main strategies: the one-to-one property (same base, set exponents equal) and the logarithm method (take log of both sides, or condense a log equation and convert to exponential form). Log equations require an extra step: always check for extraneous roots.

$$b^x = b^y \iff x = y$$

$$\log_b(x) = \log_b(y) \iff x = y$$

Exp.	Same base → one-to-one property; different base → take log/ln of both sides
Log	Condense to one log → convert to exponential form → solve → check
Check	Every log-equation solution must keep ALL original arguments positive

Solving Step Patterns

- Exponential: (1) isolate the exponential (2) take log/ln of both sides (3) apply power rule (4) divide to isolate x
- Logarithmic: (1) condense all logs to one side (2) apply product/quotient rules (3) convert to exponential form (4) solve and check for extraneous roots
- Prefer ln when the base is e; prefer $\log_{10}$ otherwise. Exponential equations never produce extraneous solutions — only logarithmic ones can.

Worked Examples

EXAMPLE 1

Solve: $4^{x+1} = 64$

- Rewrite 64 as a power of 4: $64 = 4^3$
- One-to-one property (same base): $x+1 = 3$
- Solve: $x = 2$. Check: $4^3 = 64$ ✓

Answer: $x = 2$

EXAMPLE 2

Solve: $5^x = 18$ (round to four decimal places)

- Bases can't be matched — take ln of both sides: $\ln(5^x)=\ln(18)$
- Power rule: $x \cdot \ln(5) = \ln(18)$
- Divide: $x = \ln(18)/\ln(5) = 2.8904/1.6094 \approx 1.7959$

Answer: $x \approx 1.7959$

EXAMPLE 3

Solve: $3 \cdot e^{2x} - 5 = 22$

- Isolate: $3 \cdot e^{2x}=27 \rightarrow e^{2x}=9$
- Take ln of both sides: $2x = \ln(9)$
- Divide: $x = \ln(9)/2 \approx 1.0986$

Answer: $x = \ln(9)/2 \approx 1.0986$

EXAMPLE 4

Solve: $\log_3(x + 4) = 2$

- Already isolated — convert to exponential form: $3^2 = x+4$
- $9 = x+4 \rightarrow x = 5$
- Check: $\log_3(9)=2$ ✓ (argument $9>0$)

Answer: $x = 5$

EXAMPLE 5

Solve: $\log_2(x) + \log_2(x - 2) = 3$

- Condense (product rule): $\log_2(x(x-2)) = 3$
- Convert to exponential form: $x(x-2) = 2^3 = 8 \rightarrow x^2-2x-8=0$
- Factor: $(x-4)(x+2)=0 \rightarrow x=4$ or $x=-2$
- Check $x=4$: $\log_2(4)+\log_2(2)=2+1=3$ ✓. Check $x=-2$: $\log_2(-2)$ undefined ✗ — extraneous

Answer: $x = 4$ ($x = -2$ is extraneous)

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 Solve: $2^{x-3} = 32$

Hint: Rewrite 32 as a power of 2. Then use the one-to-one property to set exponents equal.

2 Solve: $7^x = 50$ (round to four decimal places)

Hint: Take ln of both sides. Apply the power rule to bring x down. Then divide by ln(7).

3 Solve: $4 \cdot e^{x+1} = 60$

Hint: Isolate e^{x+1} first by dividing by 4. Then take ln of both sides.

4 Solve: $\log_5(2x - 1) = 2$

Hint: Convert directly to exponential form: $5^2 = 2x-1$. Then solve for x and check.

5 Solve: $\log(x + 3) + \log(x - 2) = 1$

Hint: Condense using the product rule. Convert to exponential form (base 10). Solve the quadratic and check both solutions.

Key Vocabulary

One-to-One (Exponential)	If $b^x = b^y$, then $x = y$. Works when both sides can be written as the same base.
One-to-One (Logarithmic)	If $\log_b(x) = \log_b(y)$, then $x = y$. Both sides must share the same base.
Logarithm Method	Taking the log of both sides of an exponential equation, then applying the power rule to bring the exponent down.
Extraneous Solution	A value that satisfies the transformed equation but not the original — any log-equation root making an argument ≤ 0.
Isolate the Exponential	Rewrite the equation so the exponential expression stands alone before taking a log.
Convert to Exponential Form	Rewriting $\log_b(x) = y$ as $b^y = x$ — used after condensing a log equation.
Condensing	Combining multiple logs into one using product/quotient/power rules before converting to exponential form.
Natural Log Method	Using ln to solve base-e equations. Since $\ln(e^x)=x$, the exponential and log cancel cleanly.

Check Your Understanding

Circle the letter of the correct answer for each question.

1 **Solve: $3^{x+2} = 27$** Multiple Choice

- **A** $x = 1$
- **B** $x = 3$
- **C** $x = 5$
- **D** $x = 9$

2 **Solve: $2^x = 10$ (to four decimal places)** Multiple Choice

- **A** $x \approx 2.3026$
- **B** $x \approx 3.3219$
- **C** $x \approx 1.0000$
- **D** $x \approx 0.3010$

3 **Solve: $\log_4(x - 1) = 3$** Multiple Choice

- **A** $x = 63$
- **B** $x = 65$
- **C** $x = 12$
- **D** $x = 4$

4 **Solve: $\ln(x) + \ln(x + 2) = \ln(8)$. Which value is the solution?** Multiple Choice

- **A** $x = 2$
- **B** $x = -4$
- **C** $x = 4$
- **D** $x = -2$

5 **Solve: $e^{3x} = 20$ (to four decimal places)** Multiple Choice

- **A** $x \approx 0.9986$
- **B** $x \approx 1.9459$
- **C** $x \approx 0.4330$
- **D** $x \approx 6.6667$

Common Mistakes to Avoid

✗ Forgetting to check for extraneous solutions in log equations — accepting $x=-2$ without verifying.	✓ Always substitute every solution back into the ORIGINAL equation. If any log argument is ≤ 0, reject it.
✗ Taking log of both sides before isolating: $\log(3\cdot2^x)=\log(24) \rightarrow 3x\cdot\log(2)=\log(24)$.	✓ Isolate first: $3\cdot2^x=24 \rightarrow 2^x=8$. Then use one-to-one or take log.
✗ Applying one-to-one when bases differ: $2^x = 3^x \rightarrow x = x$.	✓ One-to-one only works with the SAME base. For different bases, take log of both sides.
✗ Converting $\log_2(x)+\log_2(x-2)=3$ by solving each log separately.	✓ Condense FIRST using the product rule, then convert the single log to exponential form.

Math Tips

★ For exponential equations: try same-base first (fastest). If impossible, take ln (base e) or $\log_{10}$ (other bases).

★ Isolate the exponential BEFORE taking a log. Extra coefficients or constants must be moved first.

★ For log equations: condense to ONE log, then convert to exponential form. Never convert multiple logs separately.

★ Always check log-equation solutions in the ORIGINAL equation — not the simplified version.

★ $\ln(e^x) = x$ and $e^{\ln x} = x$. These cancellations make base-e equations especially clean.